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Note

Sedimentation focusing field-flow fractionation in channels of triangular cross-section

STANISLAV WIČAR
Hvězdárenská 3, 61600 Brno (Czechoslovakia)
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Sedimentation focusing field-flow fractionation in channels with modulated cross-sectional permeability has been proposed by Janča and Jáhnová[1]. Later Janča and Chemelík[2] published some preliminary results obtained in a gravitational field.

A limiting case of focusing based on Archimedes' forces is focusing in a pseudo-discontinuous density field. Let us consider a channel of triangular cross-section containing three layers of liquids differing only slightly in their densities. The lower part of the channel is occupied by the most dense, the middle part by a less dense and the upper part by the least dense liquid. The density gradients necessary for focusing are concentrated in the diffusional zones around the interfaces separating homogenous liquids. If a binary mixture of particles differing merely in density is introduced at the channel inlet, then after relaxation both particle species are focused at the interfaces, provided that the particle densities match those of liquid layers.

The initial particle distribution could be expressed in terms of spatial resolution, R_y:

$$R_y = \frac{y_1 - y_0}{\overline{W}} \tag{1}$$

where y_1 and y_0 are the coordinates of the two interfaces and $\overline{W}$ is the effective thickness of the interface zone. As the thickness of each diffuse zone does not depend on the medium layer height, $y_1 - y_0$, the initial spatial resolution could be generated over wide limits.

During the subsequent field-flow fractionation process the spatial resolution, R_y, is transformed into a time-based resolution, R_t. The efficiency of this process, R_t/R_y, is controlled by convective diffusion within both interface zones. To evaluate the efficiency, we have first to describe the hydrodynamics in a channel of triangular cross-section.

Janča and Jáhnová[1] modified Takahashi and Gill's[3] approximate solution of the Poisson equation for fully developed laminar flow in rectangular channels:

$$u(x,y) = \frac{\Delta P b^2}{2\eta L}\,(1 - y^2/b^2)\left[1 - \frac{\cosh(\sqrt{3}\ x/b)}{\cosh(\sqrt{3}\ w/b)}\right] \tag{2}$$

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by multiplying the right-hand side of eqn. 2 by the function

$$\left[1 + x/w\left(\frac{y_2 - y_1}{y_2 + y_1}\right)\right]^2 \tag{3}$$

to satisfy the new boundary problem for a trapezoidal cross-section. In eqn. 2 $\Delta P/L$ is the pressure drop applied to the channel, η is fluid viscosity and b and w are the channel dimensions. In expression 3 y_2 and y_1 are the channel widths at both sides of the channel.

Unfortunately, the resulting $u(x,y)$ function does not satisfy the basic Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{\eta L} \tag{4}$$

To find the solution of eqn. 4 for a channel of triangular cross-section, we may start with the construction of a function f(x,y) that vanishes at the channel walls (Fig. 1):

$$\mathrm{f}(x,y) = (y - ax)(y + ax) \tag{5}$$

where

$$a = \tan\alpha = \mathrm{cotan}\,\beta/2 = \frac{\sin\beta}{1-\cos\beta}$$

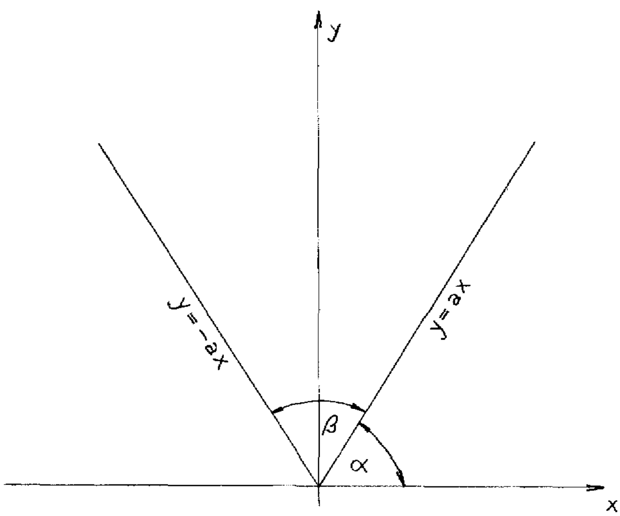

Fig. 1. Orientation of the channel in the rectangular coordinate system.

As the Laplacean of f(x,y) equals a constant:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -2(a^2 - 1)$$

we may use function 5 to form the solution of eqn. 4:

$$u(x,y) = \frac{\Delta P}{2\eta L(a^2 - 1)} (y^2 - a^2x^2) \tag{6}$$

Eqn. 6 expresses the velocity profiles in an open channel of triangular cross-section, provided that $\beta < \pi/2$. For a closed channel, the profiles are modified merely at the upper wall. The important central part of the profiles in eqn. 6 for $x = 0$ is parabolic and does not display any local minimum as the equation of Janča and Jáhnová[1] does.

The convective diffusion could be characterized by an HETP equation:

$$H = \frac{2D}{\bar{u}} + \frac{w^2\bar{u}}{\kappa D} \tag{7}$$

where w is the channel width, κ, according to Aris[4], is a dimensionless number, D is the diffusion coefficient of separated particles and $\bar{u}$ is the linear mean velocity.

For the mean velocity of each interface layer we obtain from eqn. 6, provided that $\eta_0 \doteq \eta_1 \doteq \eta_2 = \eta$,

$$\bar{u}(y) = \frac{K}{a^2 - 1} \cdot y^2 \tag{8}$$

where $K = \Delta P/3\eta L$ is the normalized pressure drop across the channel. By inserting from eqn. 8 into eqn. 7, taking into account that $w = 2y/a$,

$$H = \frac{2D(a^2 - 1)}{Ky^2} + \frac{4Ky^4}{a^2(a^2 - 1)\,\kappa D} \tag{9}$$

We may minimize H with respect to the normalized pressure drop K to obtain

$$K_m = \frac{\tau a(a^2 - 1)\,D}{y^3}$$

where $\tau = \sqrt{\kappa/2}$. Relating the minimum H to the first interface at y_0, we have

$$H_{m0} = \frac{4y_0}{a\tau}$$

and

$$\sigma_{t0} = \frac{\sqrt{H_{m0}\, L}}{\bar{u}_0} = \frac{1}{aD}\sqrt{\frac{4y_0^3\, L}{a\tau^3}} \tag{10}$$

For H_1 and the time variance of the particle distribution at the second interface, y_1, we obtain

$$H_1 = \frac{2y_1}{a\tau}\left(\frac{y_0^3}{y_1^3} + \frac{y_1^3}{y_0^3}\right)$$

and

$$\sigma_{t1} = \frac{1}{aD}\sqrt{\frac{2L}{a\tau^3} \cdot \frac{y_0^6}{y_1^3}\left(\frac{y_0^3}{y_1^3} + \frac{y_1^3}{y_0^3}\right)} \tag{11}$$

The time-based resolution at the end of the channel is by definition

$$R_t = \frac{2L(1/\bar{u}_0 - 1/\bar{u}_1)}{4(\sigma_{t0} + \sigma_{t1})}$$

and after inserting from eqns. 8, 10 and 11 we obtain

$$R_t = f(y_0,y_1)\sqrt{aL\sqrt{\kappa/2}}(y_1 - y_0) \tag{12}$$

where

$$f(y_0,y_1) = \frac{1}{y_1^2} \cdot \frac{y_0(y_1 + y_0)}{2\sqrt{y_0^3} + \sqrt{2\frac{y_0^6}{y_1^3}\left(\frac{y_0^3}{y_1^3} + \frac{y_1^3}{y_0^3}\right)}}$$

Janča and Jáhnová[1] used a channel with $\beta = 2^0$ and a = 57.3; the length of the channel was 30 cm. For $y_0 = 1$ cm, $y_1 = 2$ cm, f(1,2) = 3/16. Taking κ according to Golay[5] as 105, we have $R_t = 20.9$.

Estimating the effective thickness of the interface zones as 0.5 mm, the efficiency of the transformation process is approximately 1. Unfortunately, if the effective diffusion coefficient of separated particles is of the order of 10^{-7} cm^2/s, the time for this separation is unrealistic:

$$\frac{L}{u_0} = \frac{Ly_0}{\tau a D} = 7.2 \cdot 10^5 \text{ s} = 200 \text{ h}$$

By increasing the optimum normalized pressure drop by a factor of 100, we obtain for the time-based resolution

$$R_{t100} = f_1(y_0,y_1)\frac{\sqrt{aL\sqrt{\kappa/2}}}{40}(y_1 - y_0) \quad (13)$$

where

$$f_1(y_0,y_1) = \frac{y_1 + y_0}{y_1^2\sqrt{2y_0}}$$

and for the same channel we have $R_{t100} = 1.4$ and the efficiency of the transformation is merely 0.1; the process requires about 2 h.

It is evidently questionable whether field-flow fractionation is the best solution for the conversion of the spatial resolution to the time-based resolution in sedimentation separations using gravitation. Apparently better results could be obtained if the sedimentation focusing process were accomplished in a burette-like vertical tube. Connecting the lower end of this tube directly to the detector, the direction of flow coincides with that of the gravitational force and, once obtained, the spatial resolution is converted to its time-based form directly with greater efficiency in a substantially shorter time. Such an arrangement, of course, could hardly be called field-flow fractionation.

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